

Multi-Periodic Oscillations in Cepheids and RR Lyrae-Type Stars

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Abstract Classical Cepheids and RR Lyrae-type stars are usually considered to be textbook examples of purely radial, strictly periodic pulsators. Not all the variables, however, conform to this simple picture. In this review I discuss different forms of multi-periodicity observed in Cepheids and RR Lyrae stars, including Blazhko effect and various types of radial and nonradial multi-mode oscillations.

1 Blazhko effect

Blazhko effect is a slow, nearly periodic modulation of the pulsation amplitude and phase. It is observed in many RR Lyrae stars, both of RRab (fundamental mode) and of RRc type (overtone pulsators). The modulation period can range from a few days to over 2500 days [44]. The phenomenon was discovered over a century ago ([4, 40]) and was the first identified departure from the "strictly periodic" paradigm.

Blazhko effect has been detected in various stellar systems, including Magellanic Clouds [48, 49], galactic bulge [44] and several globular clusters (e.g [30]). By no mean it is a rare phenomenon. Modulation occurs in $\sim 50\%$ of the field RRab stars ([18, 2]). The incidence rates reported in other RR Lyrae populations are up to 4 times lower, perhaps because of lower quality of the available data. In most stellar systems the incidence rate of Blazhko modulation is higher in RRab stars than in RRc stars. The opposite is true only in the globular cluster Omega Centauri, where unusually high fraction (38%) of modulated RRc stars has been found [30].

Blazhko effect is usually associated with the RR Lyrae variables, but it can also occur (albeit rarely) in classical Cepheids. A modulation with a period of ~ 1200 days is observed in a single-mode overtone Cepheid V473 Lyr [7]. The amplitude and phase modulation has also been recently discovered in a number of LMC double-mode Cepheids [28]. These latter variables will be discussed in Sec. 2.2.

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After 100 years since its discovery, the Blazhko effect remains an unsolved mystery. Several models have been proposed to explain the phenomenon (e.g. [41, 34, 50]), but all of them have failed (see excellent review by Kovács [24]). Recent discovery of the period doubling in the Blazhko RR Lyrae stars [51, 52] offers a new hope of understanding the Blazhko enigma. The period doubling is a resonant phenomenon [26]. In the RRab models it has been traced to the 9:2 resonance between the fundamental mode and the 9th radial overtone [21]. In principle, such a coupling is capable of producing not only stationary, but also modulated solutions [6]. This makes the 9:2 resonance interaction a very promising idea, that might explain the Blazhko phenomenon. But we are not there yet. So far, nonlinear RR Lyrae models have been able to reproduce the period doubling, but without the modulation [21]. It remains to be seen if *modulated* period doubling solutions can also be found.

2 Multi-mode radial pulsators

2.1 Double-mode RR Lyrae-type star (RRd stars)

AQ Leo, the first double-mode RR Lyrae-type variable, was identified only in 1977 [16]. Since then, double-mode RR Lyrae pulsators have been identified not only in the galactic field, but also in several globular clusters (e.g. [10, 55, 54]) and in many dwarf spheroidal galaxies (e.g. [3, 9, 11, 19, 23]). 3 such stars are also known in M31 [39]. By far the largest populations of RRd stars has been found in the Magellanic Clouds: 986 variables in the LMC [48] and 258 variables in the SMC [49]. For comparison, in the Galaxy we know only about 90 RRd stars in the field [56] and 80 in the bulge [44]. Typically, double-mode pulsators constitute several per cent of the RR Lyrae star population, but in some stellar systems this fraction can be as high as 20 – 30% [9, 54] or as low as 0.5 – 1% [53, 44].

In vast majority of the RRd stars the period ratio of the two excited modes, P_1/P_0 , is in a narrow range of 0.742 – 0.748. This identifies the two modes as the first radial overtone (1O) and the radial fundamental mode (F). In most cases, the first overtone has larger amplitude than the fundamental (see e.g. [48]). When plotted on the P_1/P_0 vs. P_0 plane (the so-called Petersen diagram), the RRd stars form a well defined sequence, with the period ratio being systematically higher at longer period (e.g. [48], their Fig. 4). The exact location of the star in this plot is determined mainly by its metallicity [37, 23]. In most cases, the observed period ratios correspond to [Fe/H] between –1.3 and –2.0. Only in the unique case of the galactic bulge, the RRd sequence extends down to $P_0 = 0.35$ day and $P_1/P_0 = 0.726$, which implies metallicities as high as [Fe/H] = –0.35 [44].

2.2 Double-mode Cepheids

First double-mode Cepheids have been discovered more than half a century ago [35, 36]. These type of pulsators come in two basic flavours: they either pulsate in the fundamental mode and the first overtone (F/1O type, $P_1/P_0 = 0.695 - 0.745$) or in the first two radial overtones (1O/2O type, $P_2/P_1 = 0.79 - 0.81$). Recently, a third type of double-mode Cepheids has been discovered [45], with the first and the third overtones simultaneously excited (1O/3O type, $P_3/P_1 = 0.677$). So far, only two such stars have been identified.

Almost 700 double-mode Cepheids are currently known. The largest populations have been found in the LMC [46, 25] (90 F/1O + 256 1O/2O + 2 1O/3O) and in the SMC [47] (59 F/1O + 215 1O/2O). For comparison, in the Galaxy we know only 24 F/1O Cepheids and 16 1O/2O Cepheids. 5 F/1O Cepheids are known in M33 [1].

The values of P_2/P_1 are almost the same for all the 1O/2O Cepheids in all stellar systems. Behaviour of P_1/P_0 is different. This period ratio becomes systematically lower as the pulsation period increases. It also differs between stellar systems, being highest in the SMC, intermediate in the LMC and lowest in the Galaxy (e.g. [28], their Fig. 1). The latter property is caused by a strong metallicity dependence of P_1/P_0 .

Blazhko effect in 1O/2O double-mode Cepheids

Analysis of MACHO and OGLE-II photometry of LMC variables has led to discovery of a Blazhko effect in 1O/2O Cepheids [28]. At least 20% of these double-mode pulsators display periodic modulations. Amplitudes and phases of both modes vary, with a common period, P_B , which is always longer than 700 day. The longest possible modulation period is not known, currently it is limited only by the length of the data. Modulation is stronger for the second overtone. The variations of the two amplitudes are anticorrelated: maximum amplitude of one mode always coincides with the minimum amplitude of the other mode.

The discovery of modulated 1O/2O Cepheids shows that the Blazhko effect and the double-mode pulsations are not mutually exclusive. It also imposes very strong constraints on any proposed theoretical model of the Blazhko effect. All three currently most popular models ([41, 34, 50]) fail to account for the properties of these stars [28]. The pattern of the modulation observed in 1O/2O Cepheids suggests that some form of energy transfer between the two modes must be involved.

2.3 Triple-mode Cepheids

Triple-mode Cepheids are extremely rare objects. Only 8 are known so far, all have been found in the Magellanic Clouds [29, 45, 47]. They come in two different flavours: they either pulsate in the fundamental mode, the first and the second overtone (F/1O/2O type; 3 in LMC + 1 in SMC) or they pulsate in the first three radial

overtones (1O/2O/3O type; 2 in LMC + 2 in SMC). All triple-mode pulsators are strongly dominated by the first overtone, amplitudes of the other two modes are at least 3 times lower.

2.4 Secondary modes in R Rab stars

Thanks to high quality photometry obtained with *Kepler* and COROT space telescopes, secondary periodicities with mmag amplitudes have been detected in many fundamental mode RR Lyrae variables. A group of 9 objects clearly stands out (Table 1). In these stars, the ratio of the secondary and the primary periods fall in a very narrow range centered on 0.59. This is exactly the expected period ratio of the second overtone and the fundamental mode, P_2/P_0 ([2]; Smolec, priv. comm). The stars of Table 1 form a new group of double-mode radial pulsators: the F/2O RR Lyrae-type variables.

Table 1 F/2O double-mode RR Lyrae-type stars

star	P_0 [day]	P_2/P_0	ref.	star	P_0 [day]	P_2/P_0	ref.
V1127 Aql	0.3560	0.5821	[8]	V354 Lyr	0.5617	0.5862	[2]
MW Lyr	0.3977	0.5884	[17]	COROT 105288363	0.5674	0.5906	[15]
COROT 101128793	0.4719	0.5837	[38]	V350 Lyr	0.5942	0.5925	[2]
V2178 Cyg	0.4868	0.5854	[2]	KIC 7021124	0.6225	0.5931	[33]
V445 Lyr	0.5129	0.5852	[2]				

2.5 Multi-mode radial pulsations: theory

Linear theory: modeling pulsation periods

Each measured period of an identified mode yields a very accurate constraint on stellar parameters. In case of multi-mode radial pulsators we have two or three such constraints, which makes these objects particularly useful for testing stellar models. In the past, analysis of double-mode Cepheids motivated the revision of stellar opacities ([42]). The observed periods of double-mode variables can be used to derive metallicities of individual objects (e.g. [1, 5]), and with additional input from either the observed colours or from the evolutionary tracks, the masses and luminosities of the stars or distances to stellar systems can also be determined (e.g. [12, 22]). With the triple-mode pulsators, we can constrain stellar parameters even further [27].

Nonlinear models

Double-mode pulsations turned out to be very resilient to hydrodynamical modeling. The first nonlinear models displaying stable full-amplitude double-mode behaviour were computed only after time dependent turbulent convection was included into the codes [20, 14]. The results of these calculations have recently been questioned by Smolec & Moskalik [43]. They have shown that double-mode solutions found by [20, 14] resulted from unphysical neglect of buoyancy effects in convectively stable layers of the models. At this point, the problem of reproducing the full-amplitude double-mode pulsations is far from being solved.

3 Nonradial modes in Cepheids

Resolved low amplitude secondary frequencies have been detected in 9% of LMC first overtone Cepheids [28]. In most cases they are found very close to the frequency of the primary (radial) mode, with $|\Delta f| < 0.13$ c/d. Similar secondary periodicities have also been found in two F/IO double-mode Cepheids. Close proximity of two frequencies cannot be reproduced with the radial modes. Therefore, the secondary frequencies in these stars must correspond to *nonradial modes*. Discovery of such modes poses a challenge to the pulsation theory, which predicts that no photometrically detectable nonradial modes should be excited in Cepheid variables [32].

4 Mysterious period ratio of $P/P_1 \sim 0.62$

Secondary modes with puzzling period ratios in the range of 0.600 – 0.645 have been detected in more than 150 Magellanic Cloud Cepheids [28, 46, 47]. When plotted on the Petersen diagram, these variables follow two (LMC) or three (SMC) well defined parallel sequences. Secondary modes with almost the same period ratios have also been detected in 13 RR Lyrae stars [31]. In both classes of stars, such modes are found only in the first overtone and in the F/IO pulsators. The observed period ratios are incompatible with excitation of two radial modes, neither in Cepheids [13], nor in RR Lyrae stars (Smolec, private comm.). Therefore, the secondary frequency must be attributed to a nonradial mode.

Kepler photometry of 4 RRc stars with $P/P_1 \sim 0.62$ revealed another intriguing feature. In all 4 objects, subharmonics of secondary frequencies have been detected. This means that the secondary oscillations display a *period doubling* [31].

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